A New Approach for Finding the Shortest path in Z-Graph Using Modifiers

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Abstract
In this paper, we transform the Z-shortest path problem (ZSPP) into a fuzzy shortest path problem (FSPP) by applying the modifiers technique proposed by Shahila Bhanu M [7]. Then the fuzzy shortest path problem can be solved using suitable existing methods.

Keywords
Fuzzy sets, Z-number, Ranking function, Triangular Z-number, Fuzzy shortest path problem (FSPP), Z-shortest path problem (ZSPP), Modifiers.

1. Introduction

Numerous application problems, modelled as mathematical programming problems often involve uncertainty. Bellman and Zadeh [1] introduced the concept of decision making in fuzzy environments. Fuzzy shortest path problem was first introduced by Dubois and Prade [2] and Okada and Gen [3] proposed an algorithm on the basis of Dijkstra's Algorithm for finding the shortest path. Various papers have been published on shortest path problem in fuzzy environment using Dijkstra's Algorithm.
A graph with Z- weighted edges is known as Z-graph. Very few authors have researched the problem of finding the shortest path in a Z- graph using various algorithms.
Z-number and their modifiers have been explored by Shahila Bhanu M and Velammal G [7] and Rani P and Velammal G [8]. This paper specifically focuses on applying modifiers to the first component in Z-valuation. Now, we convert the ZSPP to FSPP using the modifier technique, and FSPP can be addressed using suitable existing methods.

2. Preliminaries

Definition 1: Graph
A graph is a pair (V, X), where V is a non-empty set, whose elements are called vertices and X is a set of unordered pairs of distinct elements of V. The elements of X are called edges. An acyclic graph is a graph without a cycle.

Definition 2: Shortest Path in Graph
Let G = (V, E) be a weighted graph, directed or undirected.
Weight of path $P = \langle v_0, v_1, v_2, \cdots, v_k \rangle$ is $w(P) = \sum w(v_{i-1}, v_i)$.
Shortest path $\delta(u, v)$ from u to v has weight

$$\delta(u, v) = \begin{cases} \min\{w(P): P \text{ is path from u to v}\} & \text{if path exists} \\ \infty & \text{Otherwise} \end{cases}$$

Definition 3: Fuzzy set
A fuzzy set 'A' on a universal set X is a function A: X → [0, 1].A is called the membership function, A(x) is called the membership grade of x in A. We write A= {(x, A(x)): x €X}
Definition 4: Formal definition of Z-number

A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in [0,1].

Definition 5: Z-weight of an edge

A Z-graph is a graph with Z-number weighted edges.

The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 6: Z-length of the path

Consider a path P from a vertex u to a vertex v, consists of edges e_1, e_2....e_n. Suppose the weights of these edges are the Z- numbers (A_1, B_1), (A_2, B_2) …..... (A_n, B_n). The Z-length of the path P is denoted by ZL (P)

ZL (P) = (A_1, B_1) (+, min) (A_2, B_2) …… (+, min) (A_n, B_n)

The shortest path problem in Z- graphs is the problem of finding a path between two specified nodes such that the Z-length of the path is minimum.

Definition 7: Operation on fuzzy numbers

Let A_1= (a, b, c), A_2= (d, e, f) be two triangular fuzzy numbers.

Then A_1+A_2 = (a+ d, b + e, c +f).

Definition 8: Binary operation on Z-numbers

Let (A_1, B_1) and (A_2, B_2) be two Z-numbers. Then

$(A_1, B_1)(+, min)(A_2, B_2) = (A_1 + A_2 , \min(B_1, B_2))$

Definition 9: Linguistic Hedge and Modifier

Linguistic hedges are special linguistic terms by which other linguistic terms are modified. Linguistic terms such as very, more or less, fairly, or extremely are examples of linguistic hedges. A linguistic hedge, H, may be applied to a fuzzy set by using a unary operation, h defined on the unit interval [0, 1].

A unary operation, which is an increasing bijection, representing a linguistic hedge are called modifier.

3. Converting ZSPP to FSPP using modifiers

Consider the statement "It is almost sure that the journey will take about 3 to 4 hours". This can be expressed in the form of Z-valuation as X isz (A, B), where X denotes the variate "journey timing", A denotes the fuzzy set "about 3 to 4 hours" and B denotes the fuzzy set "almost sure".

Now, if we want to rewrite the statement as "It is very sure that journey will take A′ ", we want to find A′ such that X isz (A′, B′) where B′ is the fuzzy set representing "very sure". This involves applying modifiers to the first component in Z-valuation. M. Shahila Bhanu has studied this problem, indicating that X isz (A, B) is considered equivalent to the statement X isz (A′, B′). So, we can replace the Z-valuation (A, B) by (A′, B′).

Consider a path P, consisting of the edges e_1, e_2, ------e_n with edge Z-weights (A_1, B_1), (A_2, B_2) …..... (A_n, B_n). The Z-length of the path P is

ZL (P) = (A_1, B_1) (+, min) (A_2, B_2) …… (+, min) (A_n, B_n)

Here, we apply modifier technique by converting all the second component equal to B.

(ie) we find A_1', A_2' ------A_n' so that $(A_1, B_1) \sim (A_1', B)$, $(A_2, B_2) \sim (A_2', B)$ -----

$(A_n, B_n) \sim (A_n', B)$

Then, ZL (P) = (A_1', B) (+, min) (A_2', B) …… (+, min) (A_n', B)

= $(A_1'+ A_2'+$ ----$A_n', B)$

Hence, it is enough to concentrate only on the first component of the edge weight.

Then by the modifier technique, the ZSPP is transformed to FSPP.

4. Modified Dijkstra's Algorithm for FSPP in a graph with fuzzy weighted edges

We establish a suitable ranking function, denoted by R for comparing two fuzzy numbers. The source vertex is represented by 's'. P is used to store all currently unvisited vertices in the graph G.

Algorithm:
Inputs:
V ={1,2,3.....n}// the set of vertices in the given connected graph G
s // the source vertex
d (i, j) for i ,j = 1,2,...n// d(i, j)= fuzzy length of arc(i, j) and d(i, j)≥ 0
Initialisation:
for every vertex i in V
{
d [i] = (∞, ∞, ∞)
Predecessor[i] =φ
}
d [s] =(0,0,0)
dmin= (∞, ∞, ∞)
Main part:

- P= V\{s}
- N=s
- While P≠ Φ
  {
- for every vertex i in P
  {
- if R(d[i]) >R(d[N] + d (i, j))
- {
- predecessor[i] =N, d[i] =d[N] + d (i, j)
- }
- if R(d[i]) <R(dmin), then next =i, dmin=d[i]

}

- N=next
- Delete N from P
- dmin= (∞, ∞, ∞)

  }

Output
d [i] // length of FSP from s to i

Example-1:
Consider a graph G in figure 1 with Z-number weighted edges.

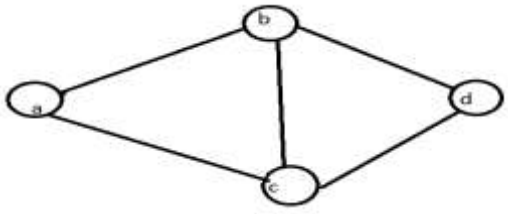


Figure - 1

We need to find the shortest path from the vertex 'a' to vertex'd'. The Z-weights of the edges are represented as follows.
w (ab) –(around 160, very likely)
w (ac) –(around 320, very likely)

w (bc) –(around 80, likely)
w (bd) –(around 490, very likely)
w (cd) –(around 320, likely)
For mathematical representation, we consider "likely" as approximately 80% (i.e), (.75, .8, .85) and "very likely" as 90% (i.e.), (.85, .9, .95).
Therefore the Z- weight of the edges is provided in the table 1.

Edges	Z-weight
Ab	((152, 160, 168), (.85, .9, .95))
Ac	((312, 320, 328), (.85, .9, .95))
Bc	((72, 80, 88), (.75, .8, .85))
Bd	((480, 490, 500), (.85, .9, .95))
Cd	((312, 320, 328), (.75, .8, .85))

Table - 1

Here, except for the edges 'bc' and 'cd', all others have the second component as "very likely". Our goal is to make the second component of all the edges equal to "very likely". To achieve this, we use the modifiers technique to ensure uniformity in the second component. According to this technique, the first component of the corresponding weight of the edges is slightly modified. Hence, we obtain a ZSPP where all parameters have "very likely" as their reliability component.
Since all the second components are equal, we have
min (the second component of all the edges) = (.85, .9, .95)
After applying the technique of modifiers to ensure uniformity in the second component, we obtain the following modified table.

Edge	Modified Z-weight
ab	((152, 160, 168), (.85, .9, .95))
ac	((312, 320, 328), (.85, .9, .95))
bc	((72, 81, 90), (.85, .9, .95))
bd	((480, 490, 500), (.85, .9, .95))
cd	((312, 321, 330), (.85, .9, .95))

Table - 2

In the modified "Z- weight table", the first component of the edges "bc" and "cd" are altered and the second components of all the vertices are made equal.
So, we concentrate only on the first component for finding SPP. Hence, the ZSPP is converted to FSPP.
We consider the ranking function as R ((x, y, z)) = $\frac{x+y+z}{3}$
Let V= {a, b, c, d}
Take s=a
Take d (a) = (0, 0, 0) and dmin= (∞, ∞, ∞)
N=a
Then P= {b, c, d}

Now d (a) + d (a, b) = (0, 0, 0) + (152, 160, 168)
= (152, 160,168)
Here, R (d (a) + d (a, b)) < R (d (b))
Then, Predecessor[b] = a, d (b) = (152, 160, 168)
Now, d (a) + d (a, c) = (0, 0, 0) + (312, 320, 328)
= (312, 320,328)
Here, R (d (a) + d (a, c)) < R (d (c))
∴ Predecessor[c] = a, d(c) = (312, 320, 328)
By ranking method, R (d (b)) < R (d (c))
So, next = b and d (b) = (152, 160, 168)
Now, N=b and P= {c. d}
d (b)+ d (b, c)= (152, 160, 168)+ (72, 81, 90)
= (224, 241,258)
Here, R (d (b) + d (b, c)) < R (d (c))
∴ Predecessor[c] =b and d(c) = (224, 241, 258)
Now, d (b) + d (b, d) = (152, 160, 168) + (480, 490, 500)
= (632, 650,668)
R (d (b) + d (b, d)) < R (d (d))
∴ Predecessor[d] = b and d (d) = (632, 650,668)
By ranking method, R (d(c)) < R (d (d))
Hence, N= c, and P= {d}
Now, d (c) + d (c, d) = (224, 241, 258) + (312, 321, 330)
= (536, 562,588)
R ((d(c) + d (c, d)) < R (d (d))
Hence d (d) = (536, 562, 588)
The shortest path in a graph with fuzzy weighted edge is a→ b→ c →d, and its length is (536, 562, 588)
Hence, the length of shortest Z- path is ((536, 562,588), (.85, .9,.95)).
So, it is **"sure"** that the shortest Z-path length is **"around 562".**

5. Conclusion

An innovative approach for solving ZSPP to FSPP by using modifier has been provided in this paper. We have demonstrated this approach by a numerical example.

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